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4D quantum black hole physics from 2D models?

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Abstract

Minimally coupled 4D scalar fields in Schwarzschild space-time are considered. Dimensional reduction to 2D leads to a well known anomaly induced effective action, which we consider here in a local form with the introduction of auxiliary fields. Boundary conditions are imposed on them in order to select the appropriate quantum states (Boulware, Unruh annd Israel-Hartle-Hawking). The stress tensor is then calculated and its comparison with the expected 4D form turns out to be unsuccessful. We also critically discuss in some detail a recent controversial result appeared in the literature on the same topic.

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Among the increasing variety of 2D dilaton gravity models, a particular attention is certainly deserved by spherically symmetric reduced General Relativity (GR) in interaction to minimally coupled massless 4D scalar fields (first considered in [1]). Because of its direct link to the reald 4D world, this model has been regarded as a reliable device to investigate four dimensional physics in the s-wave sector and not just as a "laboratory" like other 2D models. The relevant effective action (or, better, a part of it) for the matter sector can be directly obtained by functional integration of the 2D conformal anomaly [2]. This action is nonlocal, as it involves the inverse of the Delambertian operator. It can be made local by the introduction of two auxiliary fields [3].

Applying a technique which has already been tested on an analogous problem in 4D [4], in this paper we will investigate vacuum polarization around a Schwarzschild black hole induced by quantum minimal scalar fields using the above mentioned effective action. Starting from the local form of it we will impose appropriate boundary conditions to the auxiliary fields in order to select the relevant quantum states, namely Boulware (vacuum polarization around a static star), Unruh (black hole evaporation) and Israel-Hartle-Hawking (thermal equilibrium). The resulting expectation values of the stress tensor will be then compared to the ones obtained by canonical quantization and Hadamard regularization (see for instance [5]). As we shall see and as expected on the basis of previous results ([6], [7] and references therein), this check dramatically fails and once again puts serious doubts on the possibility of inferring the actual 4D behaviour starting from this lower dimensional effective action. While a similar procedure has been discussed in [3] for the Israel-Hartle-Hawking case, here the analysis is more general and extended to Boulware and, in the most nontrivial case, to the Unruh state.

We shall consider a $4D$ minimally coupled scalar field f described by the action

$$S_M^{(4)} = -\frac{1}{(4\pi)^2}\int d^4x\sqrt{-g^{(4)}}\partial^\mu f \partial_\mu f \ . \tag{1}$$

We then impose spherical symmetry both on the metric ($2+2$ splitting) and on the field f, i.e.

$$ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta = g_{ab}^{(2)}dx^a dx^b + e^{-2\phi(x^a)}d\Omega^2 \tag{2}$$

and $f = f(x^a)$ with $a, b = 1, 2$. By this ansatz the action (1) becomes

$$S_M^{(2)} = -\frac{1}{4\pi}\int d^2x\sqrt{-g^{(2)}}e^{-2\phi}\partial^a f \partial_a f \ . \tag{3}$$

From the 2D point of view this action describes a theory for a 2D massless scalar field $f(x^a)$ coupled not only to the 2D geometry $g_{ab}^{(2)}(x^a)$ but also to a "dilaton field" $\phi(x^a)$ which is related to the radius of the transverse 2D sphere (see eq. (2)). Note that the action $S_M^{(2)}$ is invariant under conformal transformations of the 2D metric $g_{ab}(x^a)$. This implies the vanishing of the trace of the corresponding classical 2D stress tensor. At the quantum level this feature is lost and we have an anomaly

$$\langle T \rangle = \frac{1}{24\pi}\left[R - 6(\nabla\phi)^2 + 6\Box\phi\right] \ . \tag{4}$$

One can functionally integrate the trace anomaly (4) to obtain the anomaly induced effective action S_{an}^{eff} for this 2D dilaton gravity model [1], [2]

$$S_{an}^{eff} = -\frac{1}{2\pi}\int d^2x\sqrt{-g}\left[\frac{1}{48}R\frac{1}{\Box}R - \frac{1}{4}(\nabla\phi)^2\frac{1}{\Box}R + \frac{1}{4}\phi R\right] \ , \tag{5}$$

where we recognize the Polyakov action besides other dilaton dependent contributions, reflecting the structure of eq. (4). One can transform this action to a local form by introducing two auxiliary fields ψ and χ [3]

$$S_{an}^{eff} = -\frac{1}{96\pi}\int d^2x\sqrt{-g}\left[2R(\psi - 6\chi) + (\nabla\psi)^2 - 12\nabla\psi\nabla\chi - 12\psi(\nabla\phi)^2 + 12R\phi\right] \ . \tag{6}$$

The auxiliary fields ψ and χ satisfy the equations

$$\begin{aligned} \Box\psi &= R \ , \\ \Box\chi &= (\nabla\phi)^2 \ , \end{aligned} \tag{7}$$

which inserted in (6) recast the action in the nonlocal form (5).

The "anomaly induced 2D stress tensor" is defined as

$$\begin{aligned} \langle T_{ab}\rangle &= \frac{2}{\sqrt{-g}}\frac{\delta S^{eff}_{an}}{\delta g^{ab}} = -\frac{1}{48\pi}\Big\{2\nabla_a\nabla_b\psi - \nabla_a\psi\nabla_b\psi - g_{ab}\left[2R - \frac{1}{2}\nabla^c\psi\nabla_c\psi\right]\Big\} \\ &- \frac{1}{4\pi}\Big\{-\frac{g_{ab}}{2}\left[(\nabla\phi)^2\psi + \nabla^c\psi\nabla_c\chi - 2(\nabla\phi)^2\right] + \nabla_a\phi\nabla_b\phi\ \psi \\ &+ \frac{1}{2}\left[\nabla_a\chi\nabla_b\psi + \nabla_b\chi\nabla_a\psi\right] - \nabla_a\nabla_b\chi\Big\} + \frac{1}{4\pi}\left(g_{ab}\Box\phi - \nabla_a\nabla_b\phi\right) \ . \end{aligned} \tag{8}$$

Our strategy will be to solve the auxiliary field equations (7) in a Schwarzschild black hole background, insert the solutions in eqs. (8) to find the 2D $\langle T_{ab}\rangle$ in the three states $|B\rangle$, $|U\rangle$ and $|H\rangle$. Here again (see [4]) the key point will be a careful choice of the arbitrary constants entering the homogeneous equations $\Box\psi = 0$, $\Box\chi = 0$.

The background is the following

$$ds^2 = g_{ab}dx^a dx^b = -(1-2M/r)dt^2 + (1-2M/r)^{-1}dr^2 \ , \ \phi = -\ln r \ . \tag{9}$$

We see that this, by eq. (2), corresponds to the 4D Schwarzschild metric. Note also that eqs. (9) extremise the gravitational action

$$S_{cl} = \frac{1}{8\pi}\int d^4x\sqrt{-g^{(4)}}R^{(4)} = \frac{1}{2\pi}\int d^2x\sqrt{-g^{(2)}}e^{-2\phi}[R^{(2)} + 2(\nabla\phi)^2 + 2e^{2\phi}] \ , \tag{10}$$

where use of the ansatz (2) has been made.

Using eqs. (9) the equations of motion for the auxiliary fields read

$$\begin{aligned} \Box\psi &= \frac{4M}{r^3} \ , \\ \Box\chi &= (1 - \frac{2M}{r})\frac{1}{r^2} \ , \end{aligned} \tag{11}$$

where $\Box$ is the Delambertian for the 2D Schwarzschild metric. The general solution of these equations that will be relevant for our purposes is

$$\begin{aligned} \psi &= C(r + 2M\ln\frac{(r-2M)}{l}) - \ln\frac{r-2M}{r} \ , \\ \chi &= bt + Dr + (2MD - \frac{1}{2})\ln\frac{(r-2M)}{l} - \frac{1}{2}\ln\frac{r}{l} \end{aligned} \tag{12}$$

where (l, C, D, b) are arbitrary constants. The presence of a linear term in t in χ has already been explained in [4]: it allows the possibility of having $\langle T_{rt}\rangle \neq 0$ but still $\partial_t\langle T_{rt}\rangle = 0$. No such term is present in ψ. A rapid look to eqs. (8) gives the reason: $\langle T_{ab}\rangle$ depends also on ψ and not just on derivatives of the auxiliary fields. Therefore $\partial_t\langle S_{ab}\rangle = 0$ requires $\partial_t\psi = 0$. Substituting eqs. (12) into (8) it is (we use Eddington-Finkelstein coordinates $\{u, v\}$ where $u = t - r - 2M\ln|r/2M - 1|$, $v = t + r + 2M\ln|r/2M - 1|$)

$$\begin{aligned} & \langle T_{uu}\rangle = -\frac{1}{48\pi}\left[\frac{2M}{r^3} - \frac{3M^2}{r^4} - \frac{C^2}{4}\right] \\ - & \frac{1}{4\pi}\Big\{\frac{1}{4r^2}(1-\frac{2M}{r})^2\left[Cr + 2MC\ln\frac{r-2M}{l} - \ln\frac{r-2M}{r}\right] \\ + & \frac{C}{16Mr^2}\left[4M(D-b)r^2 - 4Mr + 4M^2\right]\Big\} \ , \\ & \langle T_{vv}\rangle = -\frac{1}{48\pi}\left[\frac{2M}{r^3} - \frac{3M^2}{r^4} - \frac{C^2}{4}\right] \\ - & \frac{1}{4\pi}\Big\{\frac{1}{4r^2}(1-\frac{2M}{r})^2\left[Cr + 2MC\ln\frac{r-2M}{l} - \ln\frac{r-2M}{r}\right] \\ + & \frac{C}{16Mr^2}\left[4M(D+b)r^2 - 4Mr + 4M^2\right]\Big\} \ , \\ & \langle T_{uv}\rangle = \frac{1}{12\pi}(1-\frac{2M}{r})\frac{M}{r^3} \ . \end{aligned} \tag{13}$$

Boundary conditions will then be imposed on ψ and χ separately to select the relevant quantum states.

The Boulware state $|B\rangle$ coincides with Minkowski vacuum asymptotically, for which $\psi = 0$ and $\chi = -\ln\frac{r}{l}$. This limit can be achieved in eqs. (12) by setting

$C = b = D = 0$ (l is an arbitrary parameter). We find

$$\begin{aligned}
\langle B|T_{uu}|B\rangle &= \langle B|T_{vv}|B\rangle = \frac{1}{24\pi}\left[-\frac{M}{r^3}+\frac{3}{2}\frac{M^2}{r^4}\right]+\frac{1}{16\pi}(1-\frac{2M}{r})^2\frac{1}{r^2}\ln(1-\frac{2M}{r}), \\
\langle B|T_{uv}|B\rangle &= \frac{1}{12\pi}(1-\frac{2M}{r})\frac{M}{r^3}\ .
\end{aligned} \tag{14}$$

The Unruh vacuum $|U\rangle$ is constructed with modes that are regular on the future event horizon; so we require both ψ and χ to be regular as $r \to 2M$, $t \to \infty$. Such a requirement sets $C = \frac{1}{2M}$ and $D - b = \frac{1}{4M}$. This latter gives $\chi \sim v$ on the future horizon. The remaining arbitrariness is eliminated by requiring $\langle T_{vv}\rangle = 0$ as $r \to \infty$, i.e. there is no incoming radiation on past null infinity. This further yields $D + b = \frac{C}{12}$. Calculation of $\langle U|T_{ab}|U\rangle$ then gives

$$\begin{aligned}
\langle U|T_{uu}|U\rangle &= (1-\frac{2M}{r})^2\Big\{(768\pi M^2)^{-1}\left(\frac{4M}{r}+\frac{12M^2}{r^2}\right) \\
&\quad -\frac{1}{16\pi r^2}\left(\ln\frac{r}{l}+\frac{r}{2M}\right)+\frac{(1-6)}{768\pi M^2}\Big\}\ , \\
\langle U|T_{vv}|U\rangle &= (1-\frac{2M}{r})^2\Big\{(768\pi M^2)^{-1}(\frac{4M}{r}+\frac{12M^2}{r^2}-5) \\
&\quad -\frac{1}{16\pi r^2}\left(\ln\frac{r}{l}+\frac{r}{2M}\right)\Big\}+\frac{5}{768\pi M^2}\ , \\
\langle U|T_{uv}|U\rangle &= \langle B|T_{uv}|B\rangle\ .
\end{aligned} \tag{15}$$

Note the regularity of $\langle U|T_{ab}|U\rangle$ on the future horizon, i.e. $\langle T_{uu}\rangle \sim (r-2M)^2$, $\langle T_{vv}\rangle$ finite and $\langle T_{uv}\rangle \sim (r-2M)$. On future null infinity the luminosity of the hole

$$L = \frac{(1-6)}{768\pi M^2} \tag{16}$$

is negative. This disappointing result was first obtained by [1] (see also [6], [7]). The Israel-Hartle-Hawking state is an equilibrium state ($b = 0$) regular both on the future and past horizons. Regularity on these surfaces of ψ and χ is obtained by $C = \frac{1}{2M}$ and $D = \frac{1}{4M}$ yielding

$$\begin{aligned}
&\langle H|T_{uu}|H\rangle = \langle H|T_{vv}|H\rangle = \langle U|T_{uu}|U\rangle\ , \\
&\langle H|T_{uv}|H\rangle = \langle B|T_{uv}|B\rangle\ .
\end{aligned} \tag{17}$$

One easily checks that $\langle H|T_{ab}|H\rangle$ is regular on both horizons, i.e. $\langle T_{uu}\rangle \sim (r-2M)^2$, $\langle T_{vv}\rangle \sim (r-2M)^2$ and $\langle T_{uv}\rangle \sim (r-2M)$. This state describes, at infinity, thermal equilibrium. However the energy density is negative ($\frac{(1-6)}{768\pi M^2}$). This and the result for the Unruh state are physically unacceptable.

This would be all the story if the model is regarded just as one among other models of 2D dilaton gravity. However, the virtue of this particular model was its 4D origin. It was hoped therefore to obtain from the 2D analysis an insight into the real $\langle T^{(4)}_{\mu\nu}\rangle$ for minimally coupled scalars in 4D Schwarzschild spacetime. The connection between 2D and 4D stress tensors is simply [1]

$$\langle T^{(4)}_{ab}\rangle = \frac{\langle T^{(2)}_{ab}\rangle}{4\pi r^2} \tag{18}$$

and furthemore the tangential pressure P is given by

$$\langle P\rangle \equiv \langle {T_\theta}^\theta\rangle = \frac{1}{8\pi r^2\sqrt{-g^{(2)}}}\frac{\delta S^{eff}_{an}}{\delta\phi} \ . \tag{19}$$

From the action (6) we derive the pression

$$\langle P\rangle = \frac{1}{64\pi^2 r^2}\left[\frac{4M}{r^3} - 2(1-\frac{2M}{r})\frac{\partial_r\psi}{r} + \frac{2\psi}{r^2}(1-\frac{4M}{r})\right] \tag{20}$$

and inserting the solution for the auxiliary field ψ corresponding to the quantum states defined

$$\langle B|{T_\theta}^\theta|B\rangle = \frac{1}{64\pi^2}\left[\frac{8M}{r^5} - \frac{2}{r^4}(1-\frac{4M}{r})\ln(1-\frac{2M}{r})\right] \ , \tag{21}$$

$$\langle H|{T_\theta}^\theta|H\rangle = \langle U|{T_\theta}^\theta|U\rangle = \frac{1}{64\pi^2}\left(\frac{8M}{r^5} - \frac{4}{r^4} + \frac{2}{r^4}(1-\frac{4M}{r})\ln\frac{r}{l}\right) \ . \tag{22}$$

So, given $\langle T_{ab}\rangle$ $(a,b = r,t)$ from the previous calculations the corresponding $\langle T^{(4)}_{ab}\rangle$ is obtained just by dividing the results by $4\pi r^2$, whereas eqs. (21) and (22) give the remaining angular components (spherical symmetry requires $\langle {T_\theta}^\theta\rangle = \langle {T_\varphi}^\varphi\rangle$).

Analytical expressions for $\langle T_{\mu\nu}\rangle$ in the states $|B\rangle$ and $|H\rangle$ are available. According to the analysis of [5], $\langle B|T_\mu{}^\nu|B\rangle$ $(\mu,\nu = t, r, \theta, \phi)$ diverges (all components) like $(r-2M)^{-2}$ on the horizons (past and future), whereas the asymptotic $(r \to \infty)$ falloff is $O(r^{-6})$. Furthemore, it has been shown that $\langle H|T_{\mu\nu}|H\rangle$ is regular on the horizons while at infinity it has the characteristic form of thermal radiation in equilibrium at the Hawking temperature $T_H = (8\pi M)^{-1}$, namely

$$\langle H|T_\mu{}^\nu|H\rangle \to \frac{\pi^2}{30} T_H^4 diag(-1, 1/3, 1/3, 1/3) \tag{23}$$

as $r \to \infty$. Taking our results for the Boulware vacuum eqs. (14), (21) we can find through the connection formulas eqs. (18), (19) the corresponding $\langle B|T^{(4)}_{\mu\nu}|B\rangle$. It is immediately seen that $\langle B|T^{(4)}_{\mu\nu}|B\rangle$ vanishes like r^{-5} for $r \to \infty$ instead of the expected r^{-6} behaviour. Furthemore the pressure diverges as $\ln(1-2M/r)$ on the horizons and not like $(r-2M)^{-2}$. So our 2D construction does not reproduce even qualitatively the 4D $\langle B|T_{\mu\nu}|B\rangle$. The situation for the Israel-Hartle-Hawking state $|H\rangle$ is much more dramatic. As discussed in [3], we see that the asymptotic behaviour of our $\langle H|T^{(4)}_{\mu\nu}|H\rangle$ is $(Mr)^{-2}$ instead of M^{-4} (see eq. (23)). Clearly, performing a spherical reduction and then quantizing is not equivalent to the reverse procedure.

For the Unruh vacuum there are no analytical estimates of $\langle U|T_{\mu\nu}|U\rangle$. Since the 4D field equation $\Box f = 0$ and hence the modes are the same for both minimal and conformal massless scalars in the Schwarzschild spacetime, one expects, because of the Bogoliubov transformation between *in* and *out* modes, an outgoing flux at infinity of the form

$$\langle U|T_\mu{}^\nu|U\rangle \to \frac{L}{4\pi r^2}\begin{pmatrix} -1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \; r \to \infty\ , \tag{24}$$

with $L \propto M^{-2}$. Moreover $\langle U|T_\mu{}^\nu|U\rangle$ is requested to be regular (in a free falling frame) on the future event horizon. In view of the previous failures it is quite amazing to see that our $\langle U|T^{(4)}_{\mu\nu}|U\rangle$ constructed from eqs. (15), (22) does indeed behave asymptotically as eq. (24) and is regular on the future horizon. Neglecting the angular modes seems not to have drastic consequences in this case. However, we still have to face the problem of the negative luminosity (see eq. (16)) predicted by the 2D theory. It has been suggested that the addition of Weyl invariant (nonlocal) terms to S^{eff}_{an} might improve the situation [1], [7], but no definitive answer exists.

At this point we should add that the authors of Ref. [8] do not share this negative feeling towards the effective action S^{eff}_{an}. On the contrary, they claim that S^{eff}_{an} does indeed lead to positive black hole luminosity in the Unruh state, given by $L = (768\pi M^2)^{-1}$, remarkably the same value predicted by the Polyakov action. To understand this result, which is in so striking disagreement with our previous discussion and existing literature ([1], [6], [7]), we shall outline its derivation. The starting point is the conservation equations the 2D $\langle T_{ab}\rangle$ has to satisfy

$$\nabla_a\langle T^{ab}\rangle + \frac{1}{\sqrt{-g}}\frac{\delta S^{eff}_{an}}{\delta\phi}\nabla^b\phi = \nabla_a\langle T^{ab}\rangle + 8\pi e^{-2\phi}\langle P\rangle\nabla^b\phi = 0\ , \tag{25}$$

where use of eq. (19) has been made. One easily recognizes [6] these to be the 4D conservation equations $\nabla^\mu\langle T^{(4)\nu}_\mu\rangle = 0$. The physical meaning of eq. (25) is clear: it links the expectation value of the pressure in a given quantum state to the expectation value of the 2D stress tensor T_{ab} in the same quantum state. The basic assumption made in [8] is that one is not legitimate to calculate $\langle T_{ab}\rangle$ by a straightforward differentiation of S^{eff}_{an} with respect to the metric g_{ab}. This because of the subtleties involved in the correct definition of the asymptotics of

the nonlocal operator (inverse Delambertian) entering S^{eff}_{an} (see (5)). Therefore $\langle T_{ab}\rangle$ has to be calculated integrating the conservation equations (25) once $\frac{\delta S^{eff}_{an}}{\delta\phi}$ is inserted into it. This functional derivative, once S^{eff}_{an} is expressed in conformal gauge $ds^2 = -e^{2\rho}dx^+dx^-$, appears to be "local" namely

$$\frac{\delta S^{eff}_{an}}{\delta\phi} = \frac{1}{2\pi}\left[\partial_+\partial_-\rho + \partial_-(\rho\partial_+\phi) + \partial_+(\rho\partial_-\phi)\right] \tag{26}$$

and therefore eq. (25) becomes

$$\partial_\mp\langle T_{\pm\pm}\rangle + \partial_\pm\langle T_{+-}\rangle - 2\partial_\pm\rho\langle T_{+-}\rangle = \frac{\partial_\pm\phi}{2\pi}\left[\partial_+\partial_-\rho + \partial_-(\rho\partial_+\phi) + \partial_+(\rho\partial_-\phi)\right] \ . \tag{27}$$

According to the authors of Ref. [8] this approach, being entirely local, bypasses the problems mentioned above. Before proceeding we note that $\frac{\delta S^{eff}_{an}}{\delta\phi}$ in an arbitrary gauge is nonlocal (see eq. (20)) and therefore is affected by the same problem of the correct definition of $\frac{1}{\Box}$. In a conformal gauge $\{x^+, x^-\}$ the nonlocality is just hidden: different choices of conformal gauges yield different expressions for $\frac{\delta S^{eff}_{an}}{\delta\phi}$ (and hence of $\langle P\rangle$) which are not related to each other by the laws of coordinate transformation (i.e. $\frac{\delta S^{eff}_{an}}{\delta\phi}$ does not transform as a scalar density, see [6]). As experience with analogous problems in the Polyakov theory has taught us, these different expressions represent just expectation values of $\langle P\rangle$ in different quantum states. Note finally that $\langle T_{+-}\rangle$ in eqs. (27) is fixed by the trace anomaly, it is state independent and transforms correctly as a tensor

$$\langle T_{+-}\rangle = -\frac{1}{12\pi}\partial_+\partial_-\rho + \frac{1}{4\pi}\left(\partial_+\partial_-\phi - \partial_+\phi\partial_-\phi\right) \ . \tag{28}$$

It is therefore clear that the choice of gauge in eq. (26) becomes crucial: what is the gauge that correctly reproduces black hole evaporation? The mathematical procedure is then simple and requires just the insertion of the pressure term in eqs. (27), then $\langle T_{\pm\pm}\rangle$ are calculated by straightforward integration once

boundary conditions appropriate to the Unruh state are imposed. The choice of gauge made in [8] is the Eddington-Finkelstein one

$$ds^2 = -(1 - 2M/r)dudv \ , \tag{29}$$

which yields the following pressure

$$\frac{e^{2\phi}}{8\pi\sqrt{-g}}\frac{\delta S^{eff}_{an}}{\delta\phi} = \langle P\rangle = \frac{1}{64\pi^2}\left[\frac{8M}{r^5} - \frac{2}{r^4}(1-\frac{4M}{r})\ln(1-\frac{2M}{r})\right] \ . \tag{30}$$

Inserting this in eq. (27) one can find $\langle T_{uu}\rangle$ by simple integration from $2M$ to r, i.e.

$$\begin{aligned}\langle T_{uu}\rangle &= -\frac{1}{48\pi}\left(\frac{2M}{r^3} - \frac{3M^2}{r^4}\right) + \frac{1}{768\pi M^2} + \frac{1}{16\pi r^2}(1-\frac{2M}{r})^2\ln(1-\frac{2M}{r}) \ , \\ &= \left[\frac{1}{768\pi M^2}\left(1 + \frac{4M}{r} + \frac{12M^2}{r^2}\right) + \frac{1}{16\pi r^2}\ln(1-\frac{2M}{r})\right](1-\frac{2M}{r})^2 \end{aligned} \tag{31}$$

from which in the limit $r \to \infty$ one obtains the announced result

$$\langle T_{uu}\rangle \to (768\pi M^2)^{-1} \ . \tag{32}$$

In our opinion this result in view of its derivation is not unexpected. The pressure $\langle P\rangle$ of eq. (30) used in the conservation equations coincides with our eq. (21) , namely the Boulware pressure. Furthemore, given the structure of the conservation equations (27), once the Boulware pressure is inserted in the r.h.s. the resulting $\langle T_{uu}\rangle$ can differ from our $\langle B|T_{uu}|B\rangle$ just by a constant, which the above integration procedure fixes to $(768\pi M^2)^{-1}$, yielding a vanishing $\langle T_{uu}\rangle$ on the horizon. We feel rather uneasy in considering the above expression as the T_{uu} in the Unruh vacuum, as claimed in [8]. From eq. (31) we see that $\langle T_{uu}\rangle$ in a free falling frame on the future event horizon diverges logarithmically. Also, the associated pressure has the same pathology. ‡ No such behaviour is

‡The action used in [8] differs by a local term $-(4\pi)^{-1}\int d^2x\sqrt{-g}(\nabla\phi)^2$ from S^{eff}_{an}. This would give an additional contribution to $\langle T_{uu}\rangle$ in eq. (31), namely $-(8\pi r^2)^{-1}(1-2M/r)^2$, which as can be seen vanishes (quadratically) on the horizon and at infinity and therefore does not influence any of our conclusions.

expected in $\langle U|T_\mu{}^\nu|U\rangle$. The price paid to have a positive luminosity, we feel, is too high.

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